Problem Set 6

Problem 1

$$d_1 = \frac{\ln(88/90) + (0.05 + \frac{1}{2}\, 0.45^2) \times 8/12}{0.45\sqrt{8/12}} = 0.2133$$

$$\phi(-d_1) \simeq 0.4129$$

$$d_2 = 0.2133 - 0.45\sqrt{8/12} = -0.1542$$

$$\phi(-d_2) \simeq 0.5517$$

$$\text{Put} = 90\, e^{-0.05 \times 8/12} \times 0.5517 - 88 \times 0.4129 = 11.69$$

Problem 2

a. $$d_1 = \frac{\ln(50/49) + (0.045 + \frac{1}{2}\, 0.38^2) \times 3/12}{0.38\sqrt{3/12}}$$

$$= 0.2605$$

$$\phi(d_1) \simeq 0.6103$$

$$d_2 = 0.2605 - 0.38\sqrt{3/12} = 0.0705$$

$$\phi(d_2) = 0.5319$$

$$\text{Call} = 50 \times 0.6103 - 49\, e^{-0.045 \times 3/12} \times 0.5319$$

$$= 4.74.$$

b. An American call option will have the same price since the stock pays no dividends and the interest rate is positive.

c. $$\text{Put} = 49e^{-0.045 \times 3/12}(1 - 0.5319) - 50(1 - 0.6103)$$
$$= 3.19$$

d. $$\text{Call} - \text{Put} = 4.74 - 3.19 = 1.55$$
$$S - Ke^{-rT} = 50 - 49e^{-0.045 \times 3/12} = 1.55.$$

Problem 3

In this case, the price is the lower bound of the call, i.e.,

$$\text{Call} = \max\left(50 - 48\, e^{-0.08 \times 6/12}, 0\right) = 3.88$$

Problem 4

Same as before

$$\text{Call} = \max\left(50 - 52\, e^{-0.05 \times 6/12}, 0\right) = 0$$

Problem 5

The put price cannot be less than its lower bound, i.e.,

$$\text{Lower bound} = \max\left(152e^{-0.06\times 9/12} - 150, 0\right) = 0$$

Therefore, any positive price will give an implied volatility.

Problem 6

a. $$d_1 = \frac{\ln(150/155) + (0.05 + \frac{1}{2}0.55^2)\times 2}{0.55\sqrt{2}} = 0.4753$$

$$N_S = 100\,\Phi(d_1) = 68.44$$

The trader needs to buy 68.44 shares.

b. $$d_2 = 0.4753 - 0.55\sqrt{2} = -0.3025$$

$$\Phi(d_2) = 0.3745$$

$$N_B = -100\times 0.3745 = -37.45$$

The trader needs to sell 37.45 bonds with face value 155.

c. If the face value of the bonds was \$100, then the trader needs to sell 1.55 times the previous number to borrow the same amount of money.

That is, $-1.55 \times 37.45 = -58$ bonds.

d. The value of the option remains the same since the total amount borrowed does not change.

Problem 7

You can verify that $\sigma = 40\%$ gives a call price of 12.39.

Problem 8

$$d_1 = \frac{\ln(5000/5000) + (0.05 - 0.02 + \tfrac{1}{2}\, 0.30^2) \times 3/12}{0.30\sqrt{3/12}}$$

$$= 0.125$$

$$\Phi(d_1) = 0.5517$$

$$d_2 = 0.125 - 0.30\sqrt{3/12} = -0.025 \qquad \Phi(d_2) = 0.4920$$

$$\text{Call} = 5000\, e^{-0.02 \times 3/12} \times 0.5517 - 5000\, e^{-0.05 \times 3/12} \times 0.4920$$
$$= 315.30$$

Problem 9

$$d_1 = \frac{\ln(2392/2300) + (0.04 - 0.01 + \tfrac{1}{2}\, 0.50^2) \times 3/12}{0.50\sqrt{3/12}}$$

$$= 0.3119$$

$$\phi(-d_1) = 0.3745$$

$$d_2 = 0.3119 - 0.50\sqrt{3/12} = 0.0619$$

$$\phi(-d_2) = 0.4721$$

$$\text{Put} = 2300\, e^{-0.04 \times 3/12} \times 0.4721 - 2392\, e^{-0.01 \times 3/12} \times 0.3745$$

$$= 131.46$$